Quarkyonic Matter and the Phase Diagram of QCD

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Abstract

Quarkyonic matter is a new phase of QCD at finite temperature and density which is distinct from the confined and de-confined phases. Its existence is unambiguously argued in the large numbers of colors limit, $N_c \to \infty$, of QCD. Hints of its existence for QCD, $N_c = 3$, are shown in lattice Monte-Carlo data and in heavy ion experiments.

1 Quarkyonic Matter and the Large N_c limit of QCD

The large N_c limit of QCD has provided numerous insights into the structure of strongly interacting matter, both in vacuum,[1], and at finite temperature.[2] The large N_c approximation allows a correct reproduction of both qualitative and semi-quantitative features of QCD. If the number of fermions is held finite as $N_c \to \infty$, then QCD in this limit is confining in vacuum. The spectrum of the confined world consists of non-interacting confined mesons and glueballs. At finite temperature, there is a first order phase transition between a confined world of glueballs and mesons and an unconfined world of gluons. The energy density, pressure and entropy are parametrically of order one in the confined world since the confined states are colorless, but in the de-confined world are of order N_c^2, corresponding to the N_c^2 gluon degrees of freedom. The latent heat of the phase transition is of order N_c^2.

As one approaches the phase transition from lower temperature, the transition is hinted at by the existence of a Hagedorn spectrum of particles, whose density accumulates as one approaches the phase transition temperature. This accumulation resolves the paradoxical situation that at large N_c, hadrons do not interact, which seems to contradict the existence of a de-confining phase transition. If there is an accumulation of states at the Hagedorn temperature, then the the high density of states very near to the transition temperature can

compensate for weak interaction strength, resulting in a strongly interacting gas of hadrons.

The results of lattice computations for QCD ($N_c = 3$) are in qualitative accord with the results at large N_c.[3] There is a rapid change at at a well defined temperature, although not a strict discontinuity when a finite number of quark flavors are included. The presence of quarks makes the order parameter for confinement not so well defined, since the order parameter is $e^{-\beta F_q}$ where inverse temperature is β and the change in the free energy of the system, F_q, is that due to the addition of a quark. In a theory with no light quarks, one can still probe the system with a heavy quark source, which corresponds to the order parameter. In such a theory, $e^{-\beta F_q} = 0$ in the confined phase because the quarks have infinite mass. In the de-confined phase, the order parameter is finite. When light quarks are included, these light quarks can form bound states with the heavy quark probes, and so the free energy need never be infinite. The presence of light quarks therefore does not allow for an order parameter for confinement, In QCD, since there is no order parameter associated with confinement, there need be no strict phase transition, and there appears not to be for realistic quark masses. There is nevertheless a quite rapid transition at a temperature of about 200 MeV, where the energy density changes by of order N_c^2, in accord with large N_c arguments.

The popular wisdom for QCD at finite temperature and density is that there is a line of cross overs, perhaps converting to a first order phase transition at high density and low temperature, that separates the confined and de-confined world. Typically plots are made as a function of temperature T and baryon chemical potential, μ. Such a hypothetical diagram is shown in Figure 1. At very high temperature and density, there may be phase transitions associated with color superconductivity, which affect transport properties of quark-gluon matter, but are not so important for bulk properties such as pressure and energy density.[4]

Unfortunately, the conventional wisdom about the phase diagram has never been explicitly verified. It is quite difficult to disentangle the assumptions built into various model computations from the hard predictions of QCD.[5] Lattice Monte-Carlo computations are extremely difficult at high baryon density.[3]

In this talk, I will review the recent results concerning the phase diagram at finite T and μ in the limit of a large number of colors.[6] In this limit, it is possible to extract model independent results. The surprise result of these considerations is that in addition to the confined and de-confined phases of QCD, there is a third phase. It turns out that the pressure and energy density of this phase behave like a gas of quarks at very high baryon density, but nevertheless is confined. Confinement is important for properties of the matter near the Fermi surface, where excitations are required to be bound into color singlets Rob Pisarski and I named this phase the quarkyonic phase, since it has properties of both high density baryonic matter and de-confined quark matter.

To understand how such a new phase of matter might come about, we need to understand that dynamical quarks do not modify the potential between heavy test quarks at large N_c. We shall first consider the case that $N_c \rightarrow \infty$ but that
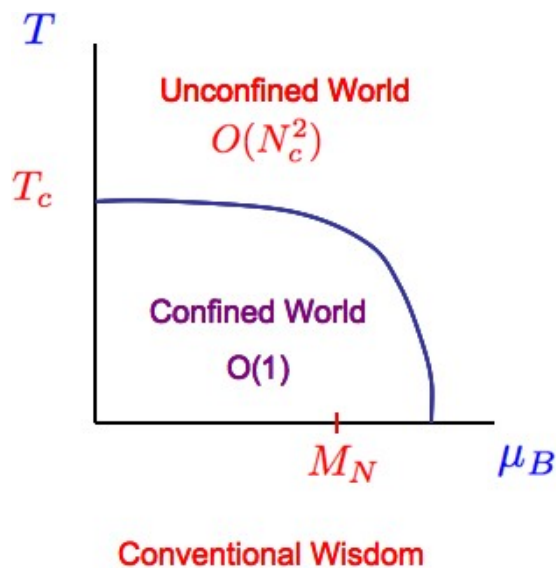
Figure 1: The conventional wisdom about the QCD phase transition at finite temperature and density.

the number of quark flavors is held fixed. We will later turn to the case where N_F/N_c is held fixed. This limit with finite number of flavors is easiest to consider since there is a confined and a de-confined phase. For finite N_F/N_c, there is no distinction between a confined and de-confined phase, although there might perhaps be a remnant of these phases associated with a cross over. In Figure 2, the gluon loop and quark loop modifications of the potential are shown. At finite temperature, the first diagram Debye screens the potential at large distances. This can short out the linear potential when the temperaute is high enough. The second diagram corresponds to a quark loop and is suppressed by $1/N_c$ at large N_c. When expressed in terms of the t'Hooft coupling, the temperature T and the quark chemical pottential μ_Q, it is of order $\alpha T^2 F(\mu_Q/T)/N_c$ Note that the baryon chemical potential is $\mu_B = N_c \mu_Q$. Typically, the baryon chemical potential is of order the baryon mass $M_B \sim N_c \Lambda_{QCD}$. The high density limit is $\mu_Q >> \Lambda_{QCD}$. In order for the quark loops to be important, the quark chemical potential must be of order $\mu_Q \sim \sqrt{N_c}\Lambda_{QCD}$ which approaches infinity in the large N_c limit

This argument shows that the presence of baryons at finite density does not affect the values of the confinement-deconfinement transition temperature. How can there be any non-trivial physics due to finite baryon number density? It turns out that at large N_c there is another order parameter, which is the baryon number density. Remember that the baryon number density is

$$\rho_B \sim e^{(\mu_B - M_B)/T} \tag{1}$$

Figure 2: Diagrams which modify the potential at high energy density .

Since both μ_B and M_B are of order N_c, $\rho_B \sim e^{-\kappa N_c}$ so long as $\mu_B \leq M_B$. Here κ is a constant of order 1. Therefore there is a finite region in the $\mu_B - T$ plane where the baryon number density is zero. This is the confined-baryonless world. At high temperature, in the de-confined world, the quarks are the correct degrees of freedom in which to measure baryon number and the baryon number is finite. As one increases the baryon number density at temperatures below the confinement temperature, there is a phase which is confined, but the baryon number chemical potential is large enough that the baryon number density is finite.

The bottom line of these arguments is that there are two order parameters corresponding to confinement and to baryon number. This in principle allows four possible phases. The phase where there is de-cofninement and zero baryon number density is apparently not realized in nature, but the other three may be.

We can compute the dependence of the pressure and energy density on N_c for the quarkyonic phase. To do this, assume the quark chemical potential is large compared to Λ_{QCD}. The pressure and energy density are computable in this limit and are of order N_c. It is remarkable that the bulk properties of quarkyonic matter may be computed using perturbation theory at very high density. This is despite that such matter is confined. This is not unprecedented, since computations of scattering processes at short distances may be done in perturbation theory, even though the processes take place in the confined vacuum. The perturbation expansion works for bulk thermodynamic quantities because the major contribution arises from quarks deep inside the Fermi sea, where short distance interactions dominate. Pairing processes near the Fermi surface are sensitive to long distance affects, and presumably pairs which form near the Fermi surface must be confined.

The fact that the energy density and pressure are proportional to N_c may also be seen in a Skyrmyonic description of baryons for two flavors. Such a description follows from the large N_c limit.[7] The action for Skyrmions is

$$S = \int d^4x \ \left(f_\pi^2 \ tr \ V^\mu V_\mu^\dagger + \kappa \ tr \ [V^\mu, V^\nu]^2 \right) \tag{2}$$

In this equation, V^μ is a derivative of a n $SU(2)$ group element, and both f_π and κ are constants of order N_c. We see that the action is therefore of order N_c, so that bulk quantities such as pressure and energy density computed in this theory should also be of order N_c

These arguments show that the confined-baryonless phase has bulk properties of order 1 in N_c, the de-coniined phase of order N_c^2 and the quarkyonic

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phase is of order N_c The phase diagram for QCD in the large N_c limit is shown in Fig. 3.
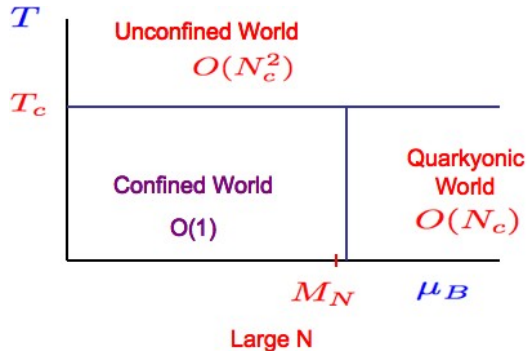

Figure 3: The phase diagram for large N_c QCD.

The change in the bulk properties of the system when μ_B crosses the confinement-quarkyonic transition is of order N_c. We can also estimate the width of the transition region. Recall that in the large N_c limit baryons are very strongly interacting. When the number density of baryons becomes of order $1/\Lambda_{QCD}^3$ or when the Fermi momentum is $k_F \sim \Lambda_{QCD}$, one will no doubt have made a transition to quarkyonc matter. This density is controlled by the Fermi momentum. For baryons with large masses of order N_c, we see that the baryon chemical potential is of order, $\mu_B \sim M_B + k_F^2/2M_B$. This means that in a width of order $1/N_c$ in μ_B or of order $1/N_c^2$ in μ_Q, the transition is achieved. In the large N_c limit, this width shrinks to zero.

2   Finite N_F/N_c

The finite ratio of N_F/N_c in the large N_c limit implies that there is no distinction between the confined and de-confined worlds at finite temperature and density. The baryon number density remains a valid order parameter. The transition is driven because of the huge degeneracy of the lowest mass baryon states. There are of order $e^{N_c G(N_F/N_c)}$ such states, where G is a function determined by Young-Tableau which counts such states. The baryon density is

$$\rho_B \sim e^{N_c G(N_F/N_c)} e^{-M_B/T + \mu_B/T} \tag{3}$$

This has a transition when $\mu_B = M_B - T N_c G(N_c/N_F)$ In this case the world is divided into that of mesons without baryons, and a world with finite baryon number density.

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3 Phenomenology and Speculation

It is tempting to speculate on the nature of the phase transition in QCD for $N_c = 3$ and realistic numbers of flavors. It is difficult a priori to know whether phase transitions remain or whether they become cross overs. I have little to say about this. It is clear that the diagram drawn in Fig. 3 becomes smoothed due to finite N_c and N_F effects. This means that the transition line at small μ_B and finite T has some small decrease as μ_B increases. When $\mu_B \sim \sqrt{N_c}\Lambda_{QCD}$, the line of finite temperature transitions will have either disappeared or have dipped near $T = 0$. If it dips down, it probably is a weak transition because the effect of de-confinement is to liberate gluons and at low T and high μ, and there the ratio of gluons to quarks is very small. Most likely, the finite temperature line of transitions ends in a second order point, if it was ever a first order transition. Presumably, there is still a tricritical region as in the large N_c limit. If the transition becomes a cross over, then perhaps a critical point slides along the line of quarkyonic-confined transition.

The large N_c considerations must be further developed in order to say anything about the chiral transition. Chiral symmetry breaking should be a Fermi surface effect in the quarkyonic phase.[8] Whether or not it is restored in the quarkyonic phase, or only approximately restored is not resolved.

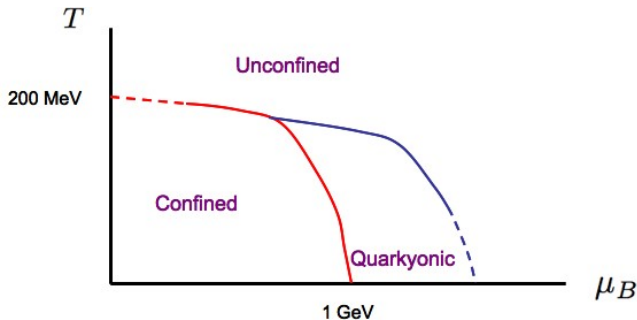A hypothetical phase diagram for QCD is shown in Fig. 4.

Figure 4: A hypothetical phase diagram for QCD.

Perhaps the strongest hints for the existence of a quarkyonic phase come from the ratios of integrated yields of particles produced in heavy ion experiments.[9]. This is supposed to give information about the decoupling temperature and chemical potential of matter produced in heavy ion collisions. Since the energy density jumps by of order $N_c N_F \Lambda_{QCD}^4$ across the quarkyonic transition, and since in large N_c, particle cross sections do not change their N_c dependence[10], one might expect that the freeze out occurs at the quarkyonic phase transition.

In Figure 5, the supposed decoupling temperature and baryon chemical potential are computed for various energies of experiments. Note that the line goes to zero temperature at about the nucleon mass. This is easy to interpret

in terms of the quarkyonic phase transition, but difficult to understand if the line corresponded to the confinement transition.
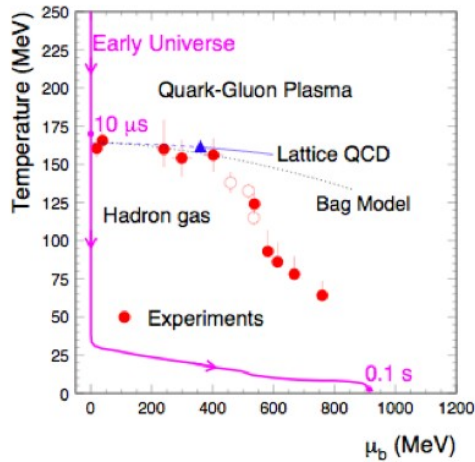

Figure 5: The decoupling temperature and density of matter produced in heavy ion collisions.

Also shown on the plot are results of computations from the bag model and from lattice gauge theory which show the weak dependence on baryon chemical potential of the confinement transition line.

There is some lattice data which argues in favor of the existence of the quarkyonic phase. In the computations of Fodor et. al., micro-canonical techniques were used on lattices of very small size.[11] They found the phase diagram shown in Figure 6.

4 Conclusions

Quarkyonic matter forces us to revise our conception of the phase diagram of QCD. There are of course many unanswered questions: How does the chiral transition interplay with the quarkyonic transition? How is quarkyonic matter related to Skyrmionic crystals?[7] What is the nature of the Fermi surface of quarkyonic matter? How is the liquid gas phase transition related to the quarkyonic phase transitions? These are of course many more.
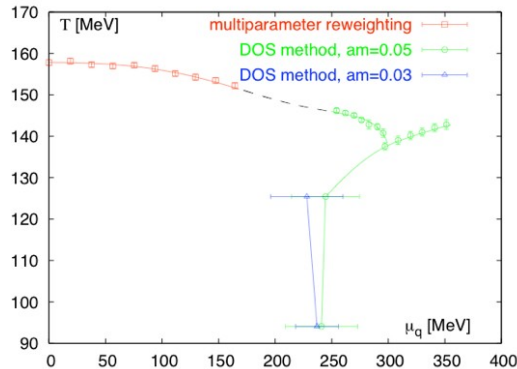
Figure 6: The phase diagram found by Fodor and colleagues.

5 Acknowledgements

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